2D Spinodal Decomposition in Forced Turbulence

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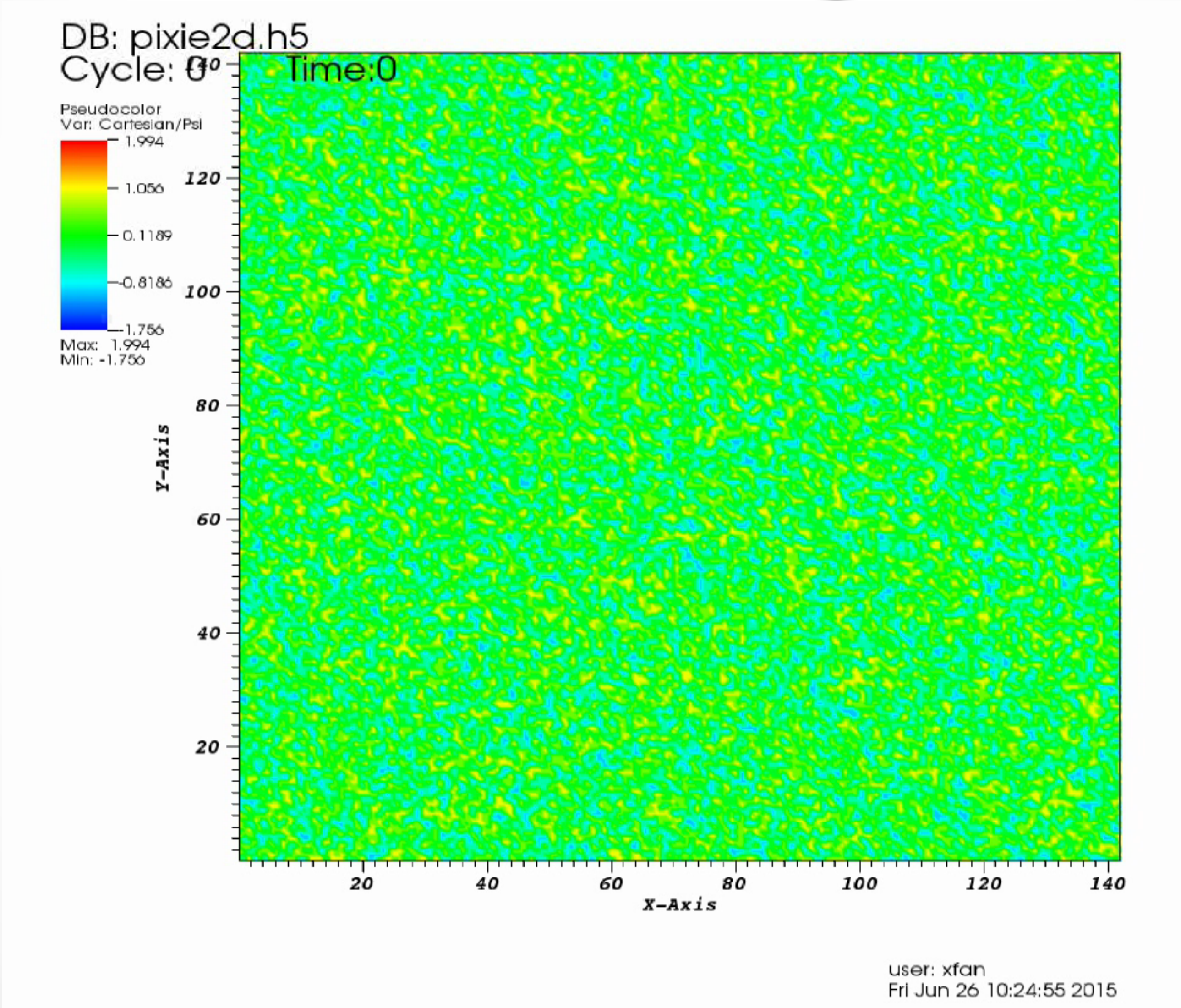
What is Spinodal Decomposition?



Cahn-Hilliard Equation

- The order parameter: the concentration:

$$\psi(\mathbf{r},t) \equiv [\rho_A(\mathbf{r},t) - \rho_B(\mathbf{r},t)]/\rho$$

- The Landau-Ginzburg Free Energy:

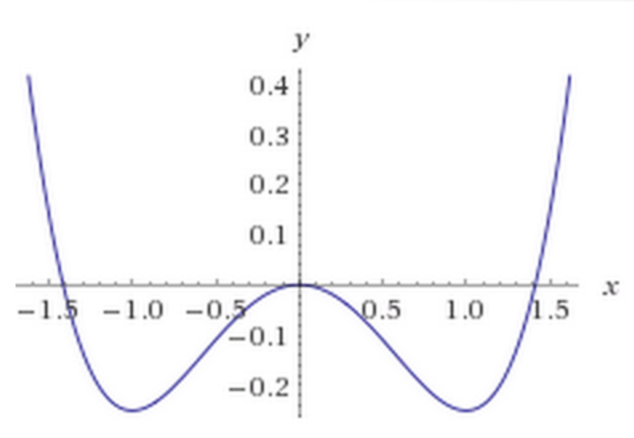
$$\Phi(\psi) = \int \mathrm{d}\mathbf{r}\left(-\frac{1}{2}\psi^2 + \frac{1}{4}\psi^4 + \frac{\xi^2}{2}|\nabla\psi|^2\right)$$

- Fick's Law:

$$\bar{J} = -D\nabla\mu$$

- Chemical Potential:

$$\mu = \frac{\delta\Phi}{\delta\psi} = -\psi + \psi^3 - \xi^2\nabla^2\psi$$



Cahn-Hilliard Equation

- Cahn-Hilliard Equation:

$$\partial_t \psi + \mathbf{v} \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

- Modified Navier–Stokes Equation:

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - \frac{\xi^2}{\rho} \nabla^2 \psi \nabla \psi + \nu \nabla^2 \mathbf{v}$$

Spinodal Decomposition vs 2D MHD

- Incompressible 2D spinodal decomposition:

$$\partial_t \psi + (\nabla \phi \times \hat{\mathbf{z}}) \cdot \nabla \psi = D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)$$

$$\frac{\partial}{\partial t} \nabla^2 \phi + (\nabla \phi \times \hat{\mathbf{z}}) \cdot \nabla \nabla^2 \phi = \frac{\xi^2}{\rho} (\nabla \psi \times \hat{\mathbf{z}}) \cdot \nabla \nabla^2 \psi + \nu \nabla^2 \nabla^2 \phi$$

- Incompressible 2D MHD:

$$\partial_t A + (\nabla \phi \times \hat{\mathbf{z}}) \cdot \nabla A = \eta \nabla^2 A$$

$$\frac{\partial}{\partial t} \nabla^2 \phi + (\nabla \phi \times \hat{\mathbf{z}}) \cdot \nabla \nabla^2 \phi = \frac{1}{\mu_0 \rho} (\nabla A \times \hat{\mathbf{z}}) \cdot \nabla \nabla^2 A + \nu \nabla^2 \nabla^2 \phi$$

Spinodal Decomposition vs 2D MHD

- Similarities:

Spinodal Decomposition	2D MHD
ψ	A
ξ^2	$\frac{1}{\mu_0}$
D	η

- Differences:
  - Three terms in chemical potential
  - $\psi \in [-1, 1]$ while A doesn't have such restriction

- So we can use theories of 2D MHD to study spinodal decomposition!

Conserved Quantities

- Energy: $E = \int (\frac{\mathbf{v}^2}{2} + \frac{\xi^2 \mathbf{B}_\psi^2}{2}) \, \mathrm{d}^2x = \int [\frac{(\nabla\phi)^2}{2} + \frac{\xi^2 (\nabla\psi)^2}{2}] \, \mathrm{d}^2x$
- Cross Helicity: $H_C = \int \mathbf{v} \cdot \mathbf{B}_\psi \, \mathrm{d}^2x = \int \nabla\phi \cdot \nabla\psi \, \mathrm{d}^2x$
- Mean Square Concentration: $C_\psi = \int \psi^2 \, \mathrm{d}^2x$
- One to one corresponding to 2D MHD conserved quantities: Energy, cross helicity, and mean square magnetic potential.
- Since negative terms in the equations, the conserved quantities do not always decay.

Conserved Quantities

$$\frac{dE}{dt} = \int (\mathbf{B}_\psi \cdot \xi^2 D \nabla^2 (-\mathbf{B}_\psi + 3\psi^2 \mathbf{B}_\psi - \xi^2 \nabla^2 \mathbf{B}_\psi))\, \mathrm{d}^2x - \nu \int \omega^2\, \mathrm{d}^2x$$

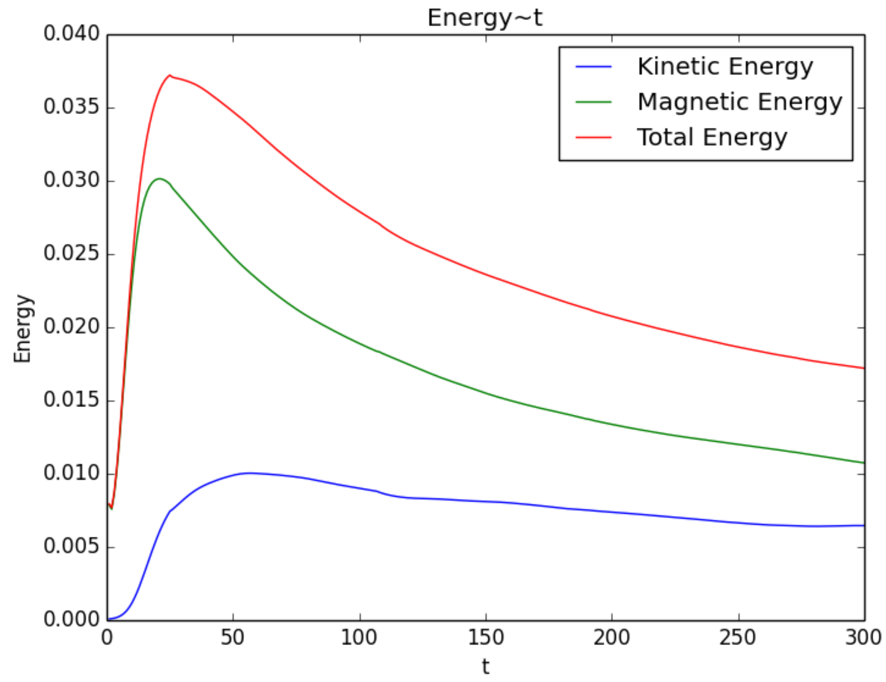
$$= -D \int (-\xi^{-2} j_\psi^2 - 6\psi B_\psi^2 j_\psi + 3\xi^{-2}\psi^2 j_\psi^2 + (\nabla j_\psi)^2)\, \mathrm{d}^2x - \nu \int \omega^2\, \mathrm{d}^2x$$

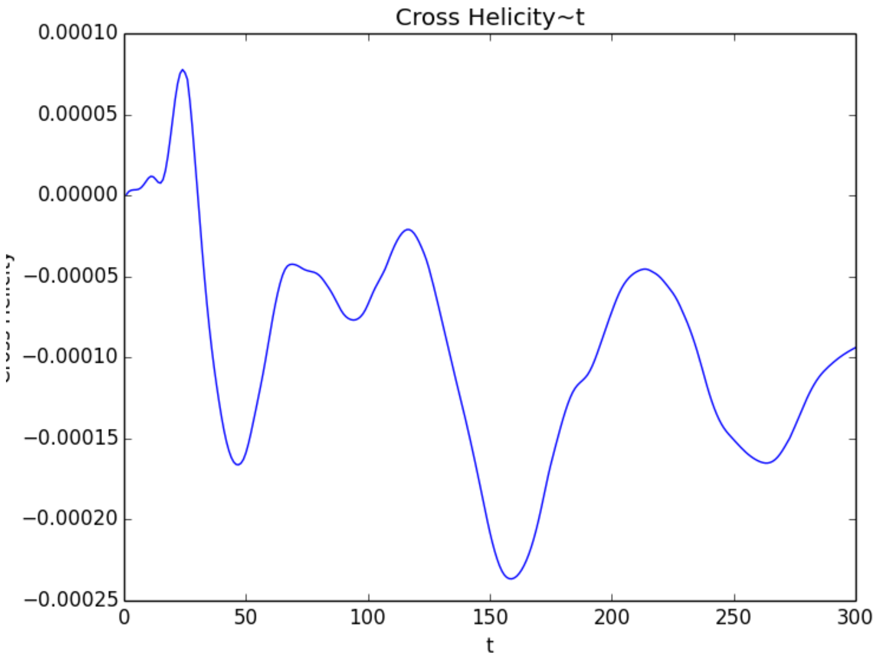
$$\frac{dH_C}{dt} = -\nu \int \xi^{-2} j_\psi \omega\, \mathrm{d}^2x - D \int (-\xi^{-2} j_\psi + 3\psi^2 \xi^{-2} j_\psi - 6\psi B_\psi^2 - \nabla^2 j_\psi)\omega\, \mathrm{d}^2x$$

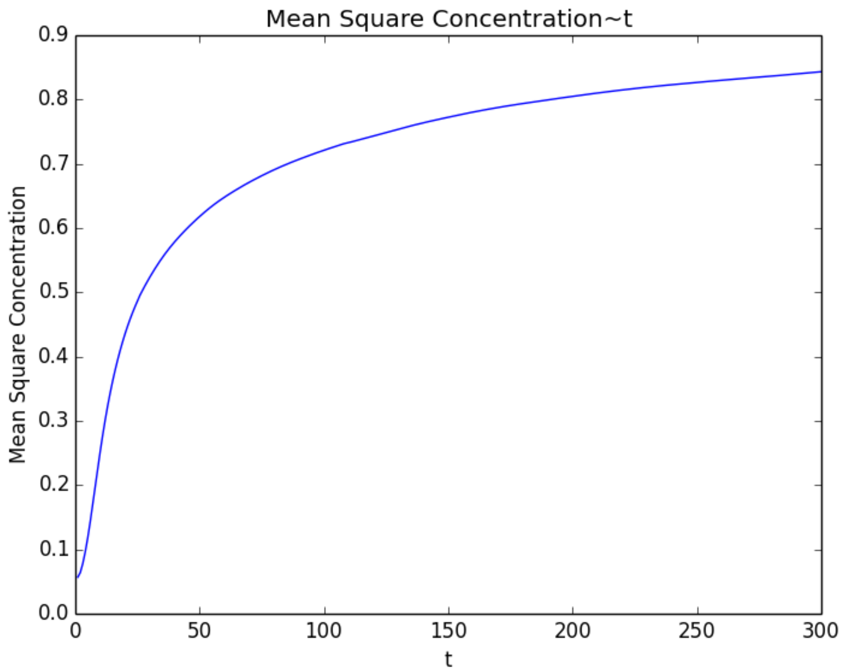
$$\frac{dC_\psi}{dt} = \int 2\psi D \nabla^2 (-\psi + \psi^3 - \xi^2 \nabla^2 \psi)\, \mathrm{d}^2x$$

$$= -2D \int [-B_\psi^2 + 6\psi^2 B_\psi^2 + \xi^{-2} j_\psi^2]\, \mathrm{d}^2x$$

Conserved Quantities

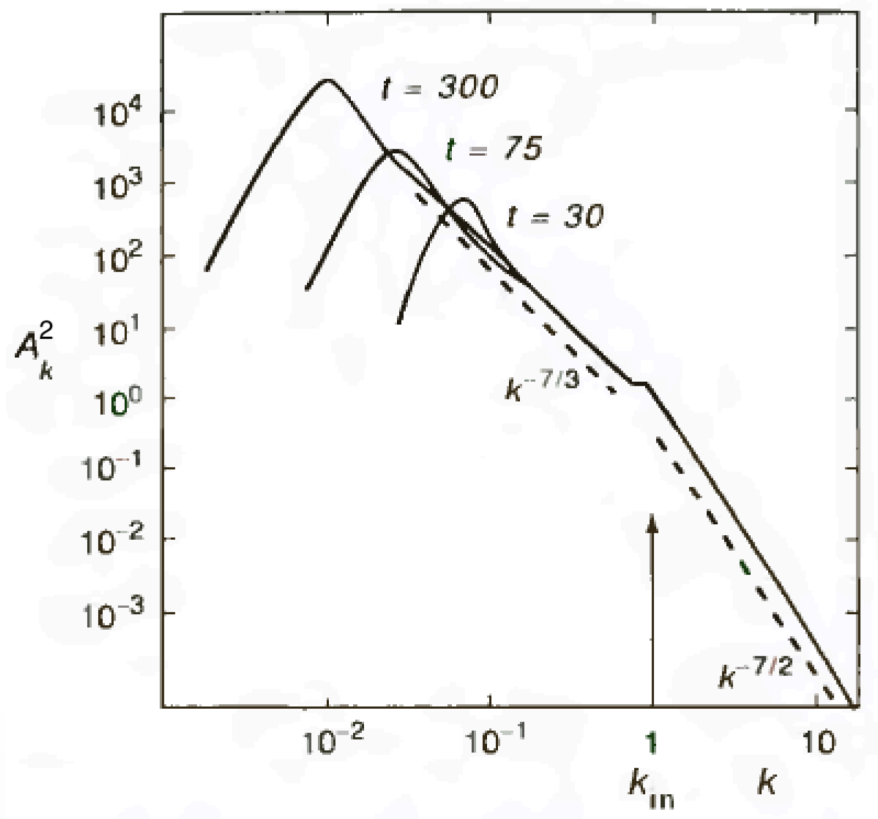






Spectra for 2D MHD

- Double cascade. (Biskamp 2003)



Spectra for 2D MHD

- Direct energy cascade: Alfven effect:

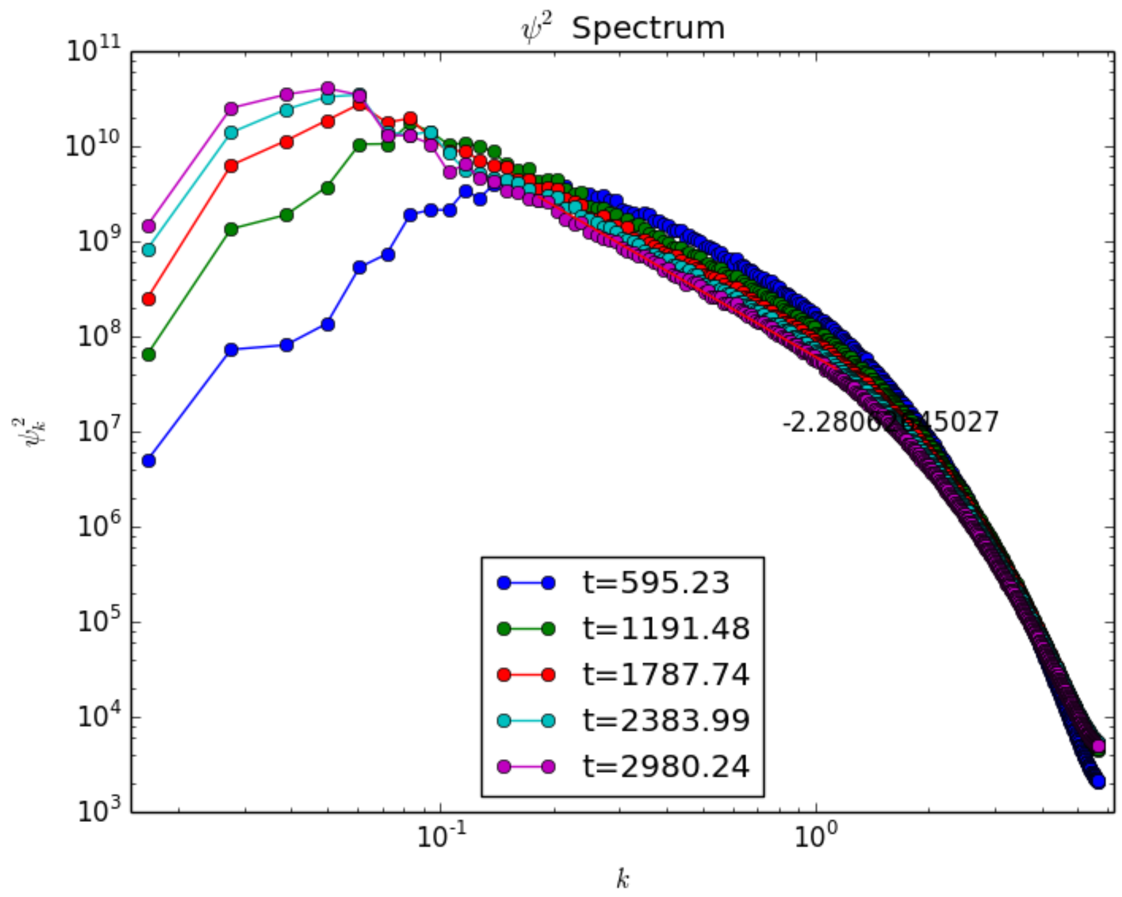
$$E_k = C_{IK}(\epsilon v_A)^{1/2} k^{-3/2}$$

$$A_k^2 = k^{-2} E_k \sim \epsilon^{1/2} k^{-7/2}$$

- Inverse A^2 cascade:

$$A_k^2 \sim \epsilon_A^{2/3} k^{-7/3}$$

Spectra for Spinodal Decomposition



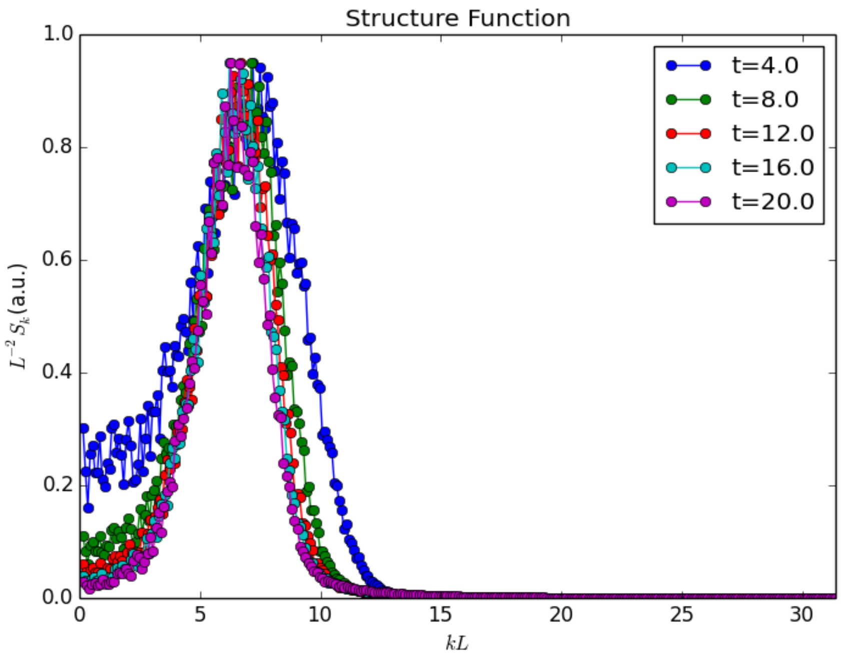
Characteristic Length Scale

Define structure function:

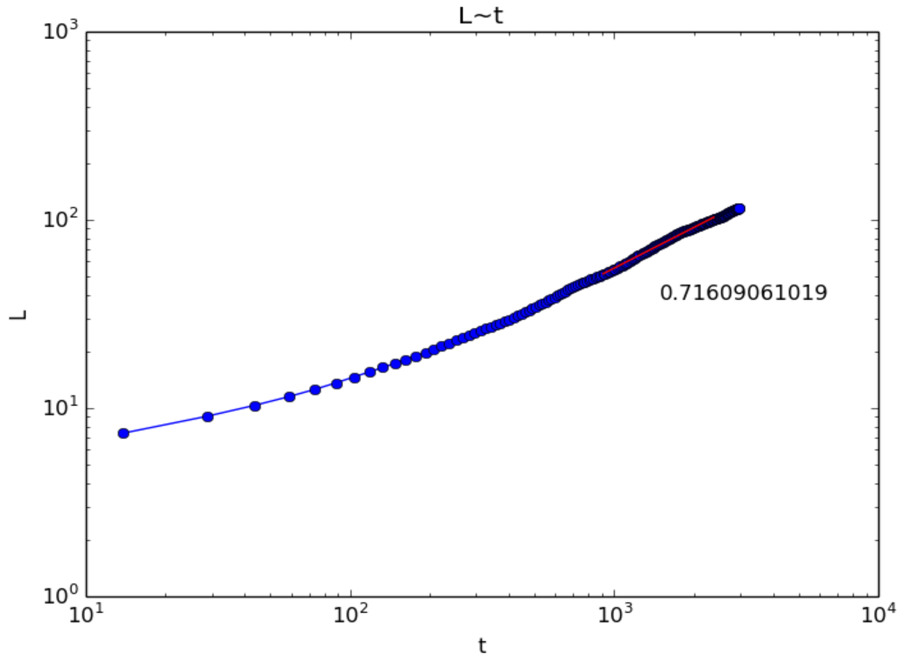
$$S_k(t) = \langle |\psi_{\mathbf{k}}(t)|^2 \rangle$$

Then the characteristic length scale is defined by:

$$L(t) = \left(\frac{\int k S_k(t)\, \mathrm{d}^2x}{\int S_k(t)\, \mathrm{d}^2x}\right)^{-1}$$



Characteristic Length Scale



- L~t^{2/3} can be derived by balancing inertial term and interfacial force term.

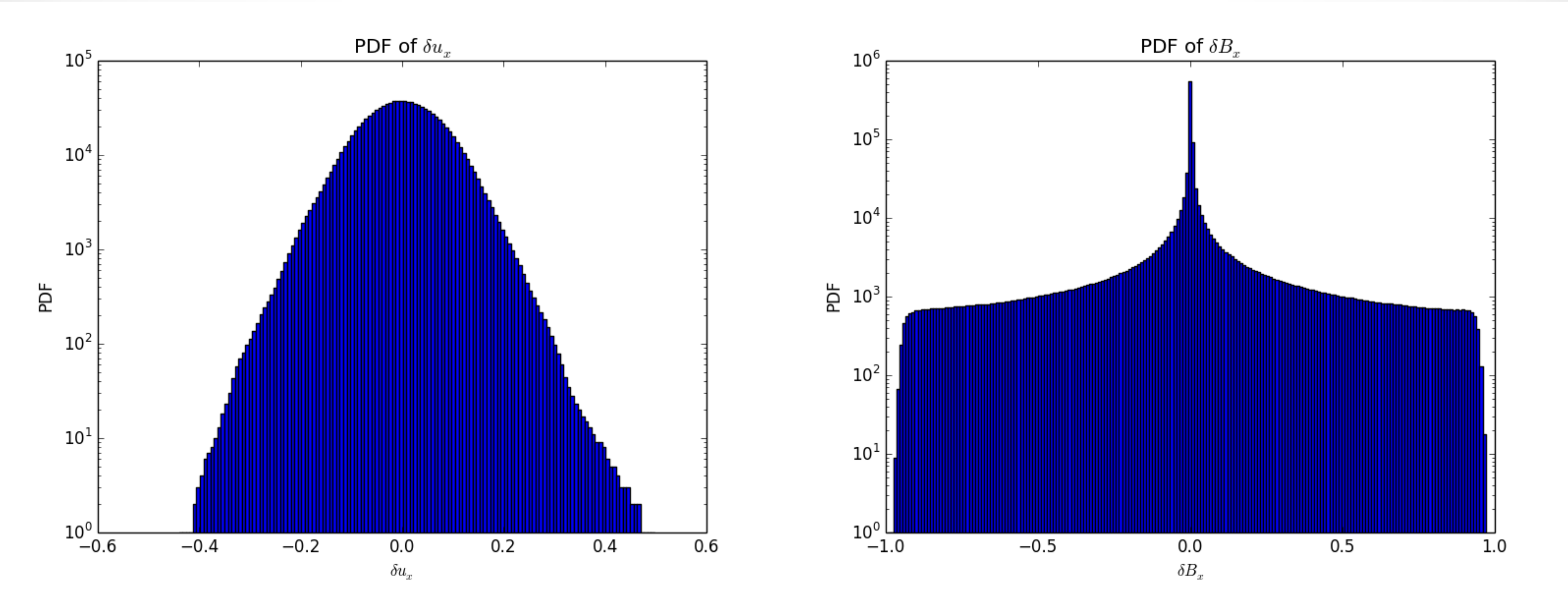
Hinze Scale

- If external forcing at large scale is present, a direct cascade will exist. This can arrest the growth of characteristic length scale.
- The final length scale will be stabilized at Hinze Scale, by balancing the turbulent kinetic energy (break up large bubbles) and surface tension energy (stick small bubbles together).

$$L_H \sim (\frac{\rho}{\sigma})^{-3/5} \epsilon^{-2/5}$$

- The competition of these two should have effects on spectra and cascades.
- Need to investigate further by simulation and theory.

Intermittency



Thanks!